Warm-up!

Let $f(x) = \frac{(x-4)(x+2)}{(3x+1)(x+2)} = \frac{x^2-2x-8}{3x^2+7x+2}$

A) What feature does the graph have at $x = -2$

B) What feature does the graph have at $x = -\frac{1}{3}$

C) What feature does the graph have at $x = 4$

Unit 1 Day 9 (1.8+1.11)

- Rational Function Zeros
- Long Division
- Crossing Horizontal/Slant Asymptotes

Homework: Zeros and Long Division

Quiz Tues/Weds (Sept 9th/10th)

What's going on at $x = c$

Let $f(x) = \frac{n(x)}{d(x)}$

If $n(c) \neq 0$ and $d(c) = 0$,
$x = c$ is a vertical asymptote

$$y = \frac{(x+3)(x-5)}{(x+3)(x+7)}$$

$x = -7$ is a vertical asymptote

If $n(c) = 0$ and $d(c) = 0$,
Simplify to find out

$$y = \frac{(x+3)(x-5)}{(x+3)(x+7)}$$

There is a hole at $(-3, -2)$

$$y = \frac{(x+3)(x-5)}{(x+3)^2(x+7)}$$

$x = -3$ is a vertical asymptote.

If $n(c) = 0$ and $d(c) \neq 0$,
$x = c$ is a zero of $f(x)$.
Graphically, this means $(c, 0)$ is an x-intercept.

$$y = \frac{(x+3)(x-5)}{(x+3)(x+7)}$$

$x = 5$ is a zero
$(5,0)$ is an x-intercept

Find all holes, x-intercepts, and vertical asymptotes for the following functions

$$f(x) = \frac{x^3 - 2x^2 + x}{9x^4 - 16x^2}$$

$$g(x) = \frac{x^3 + 6x^2 + 9x}{5x^2 + 13x - 6}$$

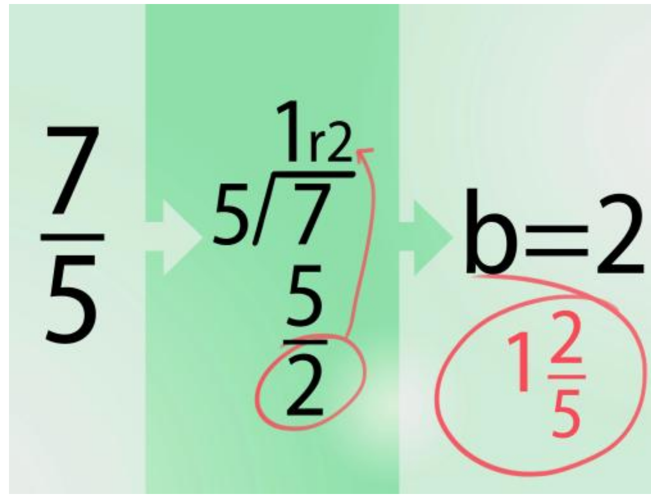
Hole vs. x-Intercept vs. Vertical Asymptote

Write a function for which $x = 4$ is a vertical asymptote, $x = 2$ and $x = -1$ are zeros, and there is a hole at $x = 3$

Polynomial (Long) Division

Divide the quotient to rewrite $f(x)$ in an equivalent form.

$$f(x) = \frac{x^2 - 8x + 23}{x - 5}$$



Polynomial (Long) Division

Divide the quotient to rewrite $g(x)$ in an equivalent form.

$$g(x) = \frac{10x - 19}{2x - 3}$$

Polynomial (Long) Division

Divide the quotient to rewrite $h(x)$ in an equivalent form.

$$h(x) = \frac{x^2 + 2x - 7}{x + 3}$$

Polynomial (Long) Division

Divide the quotient to rewrite $k(x)$ in an equivalent form.

$$k(x) = \frac{3x^2 - 17x - 23}{x^2 - 6x - 7}$$

Polynomial (Long) Division

Determine the equation of the slant asymptote.

$$y = \frac{-2x^3 + 5x^2 - 5x - 3}{x^2 - 2x + 1}$$

$$f(x) = \frac{x^2 - 3x + 2}{x + 3}$$

Vertical asymptote(s):

Horizontal asymptote:

Slant asymptote:

Hole(s):

x-intercept(s):

y-intercept:

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

$$h(x) = \frac{-(x+4)(x-6)}{x^2-9} = \frac{-x^2+2x+24}{x^2-9}$$

Vertical asymptote(s):

Horizontal asymptote:

Slant asymptote:

Hole(s):

x-intercept(s):

y-intercept:

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

$$k(x) = \frac{x+5}{x^2+4x+8}$$

Vertical asymptote(s):

Horizontal asymptote:

Slant asymptote:

Hole(s):

x-intercept(s):

y-intercept:

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

$$g(x) = \frac{3x^2 - 5x - 2}{x - 2}$$

x-intercept(s):

Vertical asymptote(s):

y-intercept:

Horizontal asymptote:

$\lim_{x \to \infty} f(x) =$

Slant asymptote:

$\lim_{x \to -\infty} f(x) =$

Hole(s):

Determine a rational function $f(x)$ with the following properties

- The only zeros are $x = 1$ and $x = 3$
- There is a hole at $x = -2$
- $\lim_{x \to \infty} f(x) = -\infty$
- $\lim_{x \to -\infty} f(x) = -\infty$